

Radiatively Driven Plasma Jets around Compact Objects

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ABSTRACT

Matter accreting onto black holes may develop shocks due to the centrifugal barrier. A part of inflowing matter in the post-shock flow is deflected along the axis in the form of jets. Post-shock flow which behaves like a Compton cloud has ‘hot’ electrons emitting high energy photons. We study the effect of these ‘hot’ photons on the outflowing matter. Radiation from this region could accelerate the outflowing matter but radiation pressure should also slow it down. We show that the radiation drag restricts the flow from attaining a very high velocity. We introduce the concept of an ‘equilibrium velocity’ ($v_{eq} \sim 0.5c$) which sets the upper limit of the terminal velocity achieved by a cold plasma due to radiation deposition force in the absence of gravity. If the injection energy is E_{in} , then we find that the terminal velocity v_{∞} satisfies a relation $v_{\infty}^2 \lesssim v_{eq}^2 + 2E_{in}$.

Key words: Astrophysical black holes, accretion, outflows, radiation drag

1 INTRODUCTION

When matter with some angular momentum is pulled in by the central gravitating object, it spirals in to form a temporary depository of matter called the accretion disc. As the accreting matter comes closer to the compact object, in much of the parameter space, the centrifugal force becomes comparable to the gravitational pull slowing down the flow considerably.

The flow may suffer a shock in a thin region (Chakrabarti, 1989; hereafter C89), where the Mach number of the flow jumps discontinuously from supersonic to subsonic value. Entropy is also generated at the shock. The region in which the flow slows down may be extended if the shock conditions are not satisfied. This hot, slowed down region puffs up in the form of a torus (hereafter, CENBOL \equiv CENtrifugal pressure dominated BOundary Layer). This disc solution which includes the advection term may be called the advective accretion disc. Chakrabarti and Titarchuk (1995, hereafter CT95) pointed out that radiation from this region is responsible for the characteristic hard and soft states of the black hole candidates. Similarly, Chakrabarti and co-workers (Chakrabarti, 1998; Chakrabarti, 1999; Das and Chakrabarti, 1999) pointed out that the same region may also generate the outflow. At the shock, a part of the hot accreting matter bounces off this effective boundary layer of the compact object, and is ejected along the axis of symmetry as outflows or jets (Das and Chakrabarti, 1999). Initially, the flow should be subsonic, since matter comes out with almost zero velocity in a very hot environment. In this region, the subsonic outflowing matter is continuously bombarded by hot photons and hence apart from usual thermal acceleration, radiative acceleration should also be important.

Interaction of radiation and matter in the context of black hole astrophysics was investigated as early as 1974 by Wickramasinghe, where he studied the radiation pressure driven mass loss from the outer most region of an accretion disc. Icke (1980) studied the effect of radiative acceleration of gas flow above a Sakura-Sunyaev Keplerian disc (1973; see also Bisnovatyi-Kogan and Blinnikov, 1977). But the effect of radiation drag on the gas flow was ignored. Sikora and Wilson (1981) showed that even if the radiation is collimated by geometrically thick discs (Lynden-Bell, 1978; Abramowicz and Piran, 1980), radiation drag is important for astrophysical jets. Piran (1982) while calculating the radiative acceleration of outflows about the rotation axis of thick accretion discs, found out that in order to accelerate outflows to $\Gamma > 1.5$ (where Γ is the bulk Lorentz factor), the funnels must be short and steep, but such funnels are found to be unstable. Sol et. al. (1989) proposed a two flow model for jets, one consists of relativistic particles (electrons and positrons) and of relativistic Lorentz factor the other being normal, mildly relativistic plasma. Melia and Königl (1989), on the other hand, considered ultra relativistic jets around super-massive black holes. These jets are assumed to be accelerated to super-relativistic Lorentz factors by hydrodynamic and electromagnetic processes close to the black hole and are Compton dragged to $\Gamma_{\infty} \leq 10$. Icke (1989) considered blobby jets about the axis of symmetry of thin discs and he obtained

the ‘magic speed’ of $v_m = 0.451c$ where c is the velocity of light. Fukue (1996) went one step ahead and found out that outflowing-rotating matter, away from the axis of symmetry, achieves terminal speed less than what Icke found previously. Sikora et. al. (1996) found out that the net radiative acceleration of AGN jets, vanishes for values of the $\Gamma_{eq} \leq 4$. Recently Fukue and his collaborators (Watarai et. al. 1999; Hirai and Fukue 2001, Fukue et. al. 2001) systematically studied radiative acceleration and collimation of winds from a disc, which is a combination of outer luminous slim disc and inner advection dominated region. In the present paper, we investigate the issue of the radiative acceleration of outflows by radiation coming out from advective accretion discs (C89; CT95; Chakrabarti, 1990a; Chakrabarti, 1996). In the papers mentioned above, in order to mimic cold plasma the pressure gradient term was altogether neglected. However, we include the pressure gradient term as well. A number of workers, such as Castor (1972), Hsieh and Spiegel (1976), Blandford and Payne (1981), Mihalas and Mihalas (1984, here after MM84), Fukue et al (1985), Kato et. al. (1998) have investigated the equations of photo-hydrodynamics. The equations of motion for non-rotating outflow is taken from MM84. We solve the governing equations of matter and radiation by the so-called ‘sonic point analysis’ (Chakrabarti, 1990b; here after C90b). The radiation energy density and flux along the axis of symmetry is calculated following the treatment of Chattopadhyay and Chakrabarti (2000).

In §2.1, we present the model assumptions. In §2.2, we present the equations of motion of the cold outflowing plasma, and in the rest of the Section we discuss the method of solution. In §3, we present the solutions, where we find that more radiation results in more acceleration, though radiation drag has a limiting effect on the terminal velocity achieved. Finally, in §4, we draw our conclusions.

2 ASSUMPTIONS, GOVERNING EQUATIONS AND THE METHOD OF SOLUTION

2.1 Assumptions

We assume that the outflows are non-viscous and non-rotating. As the astrophysical jets or outflows are observed to be extremely collimated (Bridle and Perley, 1984), the flow geometry is considered to be thin and conical. Thus the transverse structure of the jet is ignored which means all the flow variables will be calculated on the axis of symmetry and will be assumed not to vary along the transverse direction.

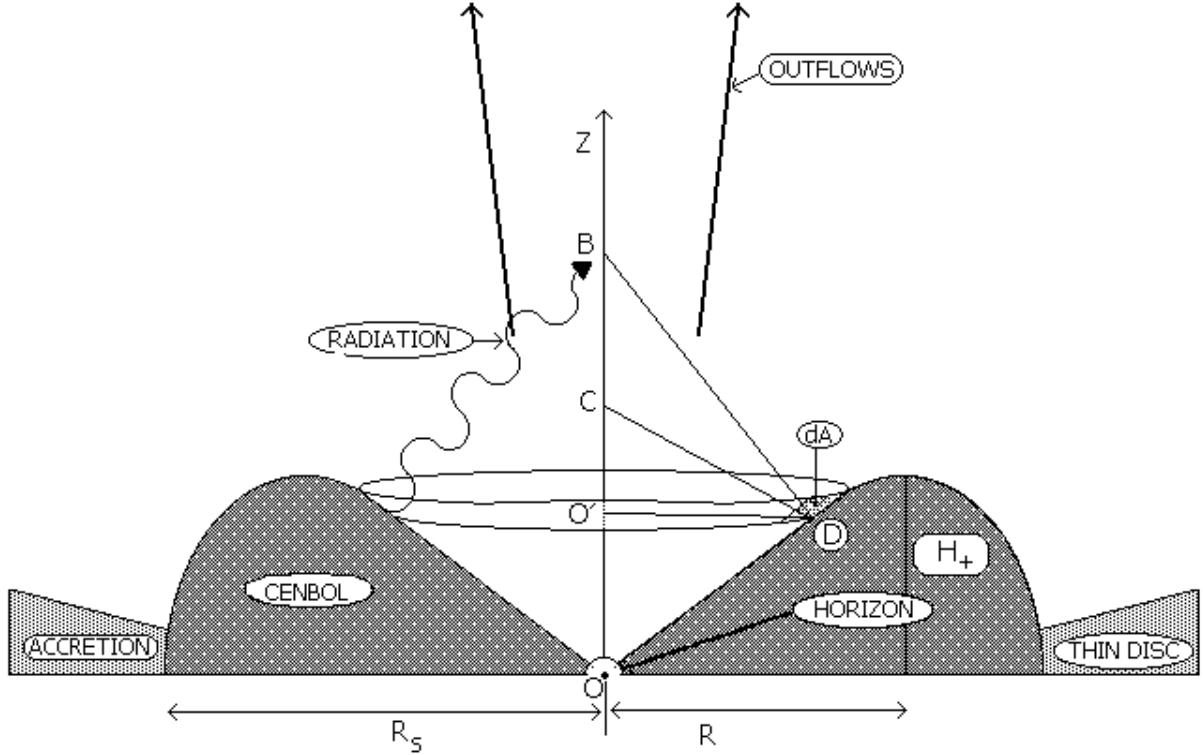


Figure 1. Schematic diagram of CENBOL and disk/outflow geometry. D is the source point, B is the field point. O is the position of compact object. The local unit normal \hat{n} is along DC . $OO' = Z'$, $O'D = X$.

In this paper we are not considering the origin of the outflowing matter self-consistently — instead, we have extracted its essential features while doing radiative-hydrodynamics. As outflows originate from the post-shock region in accretion, its initial velocity has to be necessarily subsonic. In the present analysis, this becomes the inner boundary condition of the outflow. The only connection with accretion considered here is the computation of the radiation flux, the energy density and the size of the base of the jet (i.e., CENBOL).

We are basically interested to investigate the issue of radiative acceleration of outflows by momentum deposition of photons to the electrons. In Compton scattering, the transfer of energy per scattering (Rybicki and Lightman, 1979) is given by,

$$\Delta e = \frac{e}{m_e c^2} (4k_B T_e - e), \quad (1)$$

where, Δe = net change of photon energy, e is the photon energy, k_B is the Boltzmann constant and T_e is the electron temperature. It is clear that if $e > 4k_B T_e$ then energy is transferred from photons to electrons. Hence for radiative acceleration it is necessary to consider flows to be ‘cooler’ compared to the radiation though this by itself does not ensure that the energy thus supplied can be converted to bulk kinetic energy of the outflow.

2.2 Governing Equations

Equations of motion for matter under the action of radiation has been extensively studied in the papers mentioned in §1. We use the equation of motion given by MM84 for radial flow of grey medium. We have assumed the radiation pressure to be equal to the energy density and the Eddington factor is of the order of one (see, MM84). To take care of the strong gravity around a static compact object we introduce the pseudo-Newtonian gravitational potential (Paczynski and Wiita, 1980). Equations governing the steady state flow, in the pseudo-Newtonian limit become,

Momentum Balance Equation:

$$v \frac{dv}{dz} + \frac{1}{\rho} \frac{dP}{dz} + \frac{1}{2(z-1)^2} = \mathcal{D}_z - 2v\mathcal{E}_z, \quad (2)$$

Baryon Number Conservation in the Outflow Equation:

$$\dot{M} = \Theta_{out} \rho v z^2, \quad (3)$$

where

$$\Theta_{out} = 2\pi \tan^2 \theta_0,$$

and *Entropy Generation Equation:*

$$v \left(\frac{d\epsilon}{dz} - \frac{P}{\rho^2} \frac{d\rho}{dz} \right) = Q_+ - Q_-. \quad (4)$$

The variables v , ρ and P in the above equations are, velocity, density and isotropic gas pressure respectively of the flow. θ_0 is the opening angle of the conical outflow. In the above equations, $\mathcal{D}_z = \frac{\sigma_T}{m_p} F_z$, where F_z is the z-component of flux of radiation on the axis of symmetry and $\mathcal{E}_z = \frac{\sigma_T}{m_p} E$, where E is the radiation energy density on the axis of symmetry. One must note, \mathcal{D}_z is the radiative acceleration term and is abbreviated as RAMOD (RAdiative MOmentum Deposition force) and the second term in RHS of Eq. (2) is the radiation drag term. \mathcal{D}_z and \mathcal{E}_z is calculated following the prescription of Chattopadhyay and Chakrabarti (2000), which is given in §2.3, for the sake of completeness. The third term on the LHS of Eq. (2) is the gravity with ‘Paczynski-Wiita’ (psuedo-Newtonian) potential. The first term (Q_+) in LHS of Eq. (4) is the absorption term or any other heating term, which is considered to be zero. The second one is the cooling (bremsstrahlung, synchrotron etc) term. We neglect cooling terms. Later in this paper, we shall come back to them. ϵ in LHS of

Eq. (4) is the specific internal energy term and $\epsilon = \frac{P}{(\gamma-1)\rho}$. As jets are extremely collimated, they are assumed to be in a narrow cone about the axis of symmetry (z-axis), moving radially outward, we choose $\theta_0 \approx 10^\circ \approx$ opening angle of the bipolar outflow, considered to be constant here. The gas is assumed to obey an ideal equation of state,

$$P = \frac{\rho k_B T}{\mu m_p}. \quad (5)$$

Here T is the temperature, m_p is the proton mass and $\mu = 0.5$. We define the adiabatic sound speed a as,

$$a^2 = \frac{\gamma P}{\rho}. \quad (6)$$

Units of mass, length and time are chosen to be M_B (Mass of the compact object), $2\frac{GM_B}{c^2}$ (Schwarzschild radius; G =Universal Gravitational constant) and $2\frac{GM_B}{c^3}$ respectively, also $c = 1$.

2.3 Calculation of D_z

Figure 1 shows the schematic diagram of a disc/jet geometry. The flux of radiation at field point $B(z)$, from differential area dA around a source point $D(X, \phi, Z')$ is given by:

$$d\mathbf{F} = I \cos \angle(CDB) d\Omega \frac{\mathbf{DB}}{|\mathbf{DB}|}, \quad (7)$$

where I is the frequency averaged radiation intensity on the surface and $d\Omega$ is the solid angle subtended by the differential area dA at B . The net momentum transferred (in dimensionless units) at B , due to the annular area on the cone, between Z' and $Z' + dZ'$, along z axis is given by:

$$d\mathcal{D}_z = \frac{2\pi I \sigma_T}{m_p} \tan^2 \theta \frac{z(z - Z')Z'}{[(z - Z')^2 + (Z' \tan \theta)^2]^2} dZ', \quad (8)$$

where, σ_T is the Thomson scattering cross section. θ is the semi-vertical angle of the inner surface of the CENBOL (see, Fig. 1) which is assumed to remain constant. The direction of $d\mathcal{D}_z$ is along z axis, as the transverse component of the momentum transferred, due to each of the differential area dA , cancels each other as we integrate over ϕ . We integrate over Z' to get the entire radiative contribution of the CENBOL, and hence \mathcal{D}_z (RAMOD). Thus the analytical expression of RAMOD along z is:

$$\mathcal{D}_z = \mathcal{D}_0 [\cos 2(\Phi_1 - \theta) - \cos 2(\Phi_2 - \theta)], \quad (9)$$

where

$$\Phi_1 = \tan^{-1} \left[\frac{2(H_+ - z \cos^2 \theta)}{z \sin 2\theta} \right],$$

and

$$\Phi_2 = \tan^{-1} \left[\frac{2(H_- - z \cos^2 \theta)}{z \sin 2\theta} \right].$$

H_+ and H_- are maximum and minimum height of the CENBOL from the equatorial plane, respectively, and

$$\mathcal{D}_0 = \frac{\pi I \sigma_T}{2m_p}. \quad (10)$$

Now, we have to make an estimate of I . We assume that the radiation coming out is the binding energy of the last stable orbit of accreting matter. Hence,

$$I = \frac{L}{\Omega_T \mathcal{A}}. \quad (11)$$

$L = \eta \dot{M}_{acc} c^2$, is the amount of rest mass energy of the accreting matter converted into radiation per unit time. Here \dot{M}_{acc} is the accretion rate. η is the conversion ratio and is equal to 0.06 for the Schwarzschild metric. Ω_T is the solid angle in which the radiation is coming out locally. \mathcal{A} is the total area of the inner surface of the CENBOL. I is assumed to be uniform here.

2.4 Calculation of \mathcal{E}_z

In reference to Fig. 1, one can also calculate the radiation energy density. The radiation energy density at the field point $B(z)$ is given by:

$$dE = \frac{I}{c} \cos \angle(CDB) d\Omega. \quad (12)$$

Integrating over the inner surface of the CENBOL as in the previous case we have:

$$E = E_0 z \left[-\frac{1}{\{(z' - z \cos^2 \theta)^2 + z^2 \sin^2 \theta \cos^2 \theta\}^{1/2}} + \frac{1}{z \sin^2 \theta} \sin \left\{ \tan^{-1} \frac{z' - z \cos^2 \theta}{z \sin \theta \cos \theta} \right\} \right]_{H_-}^{H_+}.$$

Hence,

$$\mathcal{E}_z = \mathcal{E}_0 z \left[-\frac{1}{\{(z' - z \cos^2 \theta)^2 + z^2 \sin^2 \theta \cos^2 \theta\}^{1/2}} + \frac{1}{z \sin^2 \theta} \sin \left\{ \tan^{-1} \frac{z' - z \cos^2 \theta}{z \sin \theta \cos \theta} \right\} \right]_{H_-}^{H_+}, \quad (13)$$

where,

$$\mathcal{E}_0 = 2\pi \sin^2 \theta \cos \theta \frac{I \sigma_T}{m_p}, \quad (14)$$

and quantities $[...]_{H_-}^{H_+}$ refers to $[...]_{H_+} - [...]_{H_-}$. We have chosen the shock in the accretion flow to be located at distance $r = R_s = 10r_g$ (see CT95) where $1r_g = 2GM_B/c^2$ and

the specific angular momentum of the accretion to be $1.6(2GM_B/c)$. This determines the geometrical structure of the CENBOL. From Fig. 1, we see that R is the position of H_+ on the equatorial plane of the accretion disc. And R_s is the maximum radial distance of the CENBOL. With the above choice of accretion parameters, we have $H_+ = 4.4r_g$ and $R = 6.671r_g$. $H_- = 1.1r_g$. With these geometric structure of CENBOL is determined which in turn helps to calculate \mathcal{D}_z and \mathcal{E}_z . Though it seems that geometric structure of CENBOL introduces additional parameters, but one must remember that these along with the total amount of radiation, are dependent on the accretion parameters. So self-consistent accretion-jet solution should determine these from the accretion parameters (namely, accretion rate and angular momentum) only.

2.5 Method

Sonic Point Analysis: To solve Eqs. (2)-(4) we use the method of sonic point analysis (C90b). In this paper the outflow is assumed inviscid and fully ionised plasma. Moreover, only electron scattering is considered. Thus, $Q_+ = 0$ in Eq. (4). The cooling terms are not considered for the time being (we will come back to them in later part of § 3), hence $Q_- = 0$. Equations (2)-(4) with the help of Eqs. (5) and (6) can be written as;

$$\frac{dv}{dz} = \frac{\frac{2a^2}{z} - \frac{1}{2(z-1)^2} + \mathcal{D}_z - 2v\mathcal{E}_z}{v - \frac{a^2}{v}} \quad (15)$$

and

$$\frac{da}{dz} = -\frac{a(\gamma-1)}{z} - \frac{a(\gamma-1)}{2v} \frac{dv}{dz}. \quad (16)$$

Since an outflow around compact objects must originate from inflowing matter, it must start with a subsonic velocity. Therefore relativistic outflows which acquire supersonic terminal velocity at large distances from the central object, must be transonic in nature. Thus, at some point (critical or sonic point), the denominator of Eq. (15) must go to zero. Since the flow must be smooth everywhere, this implies that at radial distance where the denominator of Eq. (15) goes to zero, the numerator must go to zero as well. Using this property we have the so-called *critical point conditions* or *sonic point conditions*. The sonic point conditions are :

$$v_c^2 = a_c^2, \quad (17)$$

and

$$a_c^2 - \mathcal{E}_{z_c} a_c + \frac{z_c}{2} \mathcal{D}_{z_c} - \frac{z_c}{4(z_c - 1)^2} = 0, \quad (18)$$

where z_c is the sonic point. By definition it is the distance on the axis of symmetry at which the sonic point conditions are satisfied. As $z \rightarrow z_c$, $\frac{dv}{dz} = \frac{N}{D} \rightarrow \frac{0}{0}$. The gradient of radial velocity at z_c is found out by l'Hôpital's rule,

$$\left(\frac{dv}{dz} \right)_c = \left[\frac{\frac{dN}{dz}}{\frac{dD}{dz}} \right]_c. \quad (19)$$

The expression of gradient of sound speed at z_c , is obtained by modifying Eq. (16),

$$\left(\frac{da}{dz} \right)_c = -\frac{a_c(\gamma - 1)}{z_c} - \frac{(\gamma - 1)}{2} \left(\frac{dv}{dz} \right)_c. \quad (20)$$

To obtain a complete transonic solution, we supply z_c , \dot{M}_{acc} and integrate Eqs. (15) and (16), with the help of Eqs. (17)-(20), once from z_c inwards and again from z_c outwards to a large distance. Once we obtain v and a as functions of z , we have a complete solution of the problem. Other flow variables like T can be found out from Eqs. (5) and (6) and ρ from Eq. (3). To get an estimate of the electron temperature, we can use the expression $T_e = (\frac{m_e}{m_p})^{1/2} T_p$. Calculation of ρ and estimation of T_e are used when cooling mechanisms are considered. To calculate ρ , one has to supply the value of \dot{M} , the only constant of motion. From the works of Chakrabarti and his collaborators, it is known that \dot{M} is not a free parameter, and can be calculated as a function of \dot{M}_{acc} and the compression ratio at the shock of the accretion disc. We make an estimation of \dot{M} from \dot{M}_{acc} , following Chakrabarti (1999).

Here the accretion and wind is assumed to be conical. According to Chakrabarti (1999),

$$\frac{\dot{M}}{\dot{M}_{acc}} = \frac{\Theta_{out}}{\Theta_{acc}} \frac{R_c}{4} \left[\frac{R_c^2}{R_c - 1} \right]^{3/2} \exp\left(\frac{3}{2} - \frac{R_c^2}{R_c - 1}\right) \quad (21)$$

where, \dot{M} is the mass outflow rate, \dot{M}_{acc} is the mass accretion rate and R_c is the compression ratio at the shock in accretion. Θ_{out} is the solid angle subtended by the outflow, Θ_{acc} is the solid angle subtended by the accreting matter. Assuming $\Theta_{out} \sim \Theta_{in}$ (for simplicity) Chakrabarti calculated the mass outflow rate for any generic value of compression ratio, the latter varying from 1 (no-shock) to 7 (very strong shock). Putting $R_c = 1$ (no-shock) in Eq. (21) Chakrabarti found out $\dot{M} = 0$, i.e., there is no outflow if there is no shock in accretion. The maximum outflow rate calculated was about 30% of the accretion (for $R_c \sim 4$). In the

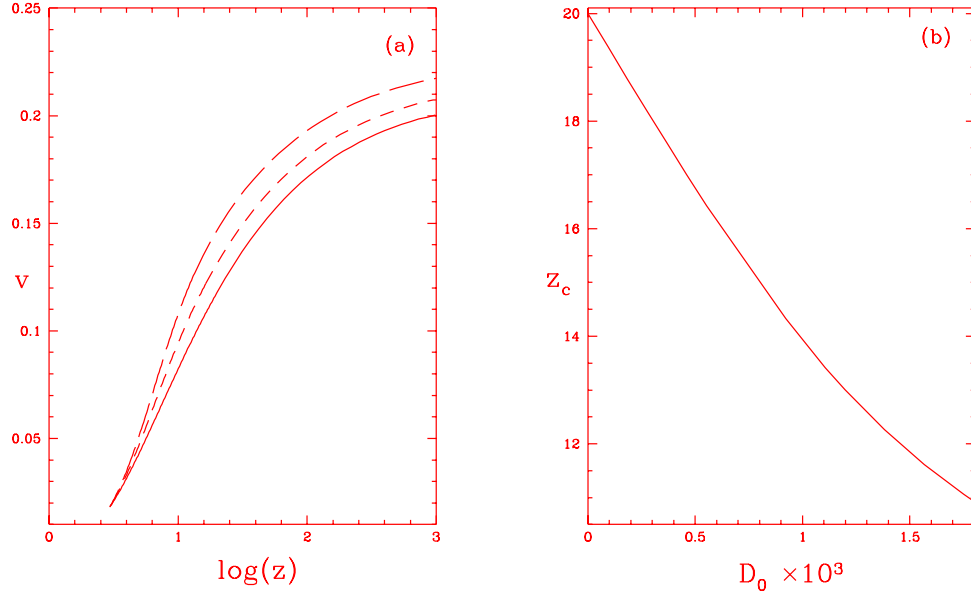


Figure 2. Solution of outflow variables plotted with $\log(z)$. Initial parameters are $z_{in} = 2.97$, $v_{in} = 0.02$. (a) Comparison of velocity variation (v) for RAMOD corresponding to $\dot{M}_{acc} = 12\dot{M}_{Edd}$ (long-dashed), $\dot{M}_{acc} = 6\dot{M}_{Edd}$ (short-dashed) and Bondi-type (solid) outflow. (b) Variation of sonic point z_c with RAMOD (D_0 is the space independent part of \mathcal{D}_r).

strong shock limit (for $R_c \sim 5$) it was calculated to be less than 10%. In the present work, the mass outflow rate is assumed to be,

$$\dot{M} = 0.01\dot{M}_{Edd}, \quad (22)$$

until otherwise stated. Of course, the actual value of mass outflow rate is necessary only while considering effects of various cooling mechanisms.

3 RESULTS

In view of RHS of Eq. (4) to be zero, Eq. (2) can be integrated and can be written in the form,

$$\frac{1}{2}v(z)^2 + na(z)^2 - \frac{1}{2(z-1)} = \frac{1}{2}v_{in}^2 + na_{in}^2 - \frac{1}{2(z_{in}-1)} + \int_{z_{in}}^z (\mathcal{D}_{z'} - 2v\mathcal{E}_{z'})dz' \quad (23a)$$

or,

$$E(z) = E_{in} + \int_{z_{in}}^z (\mathcal{D}_{z'} - 2v\mathcal{E}_{z'})dz' \quad (23b)$$

Equation (23) shows that, in absence of radiation, non-rotating outflows are accelerated by thermal pressure only and such outflows are called Bondi-type (1952) outflows. From the above equation, it is quite clear that in order to accelerate outflows *significantly* by radiation, the work done by radiation forces, onto the flow has to be *comparable* to the

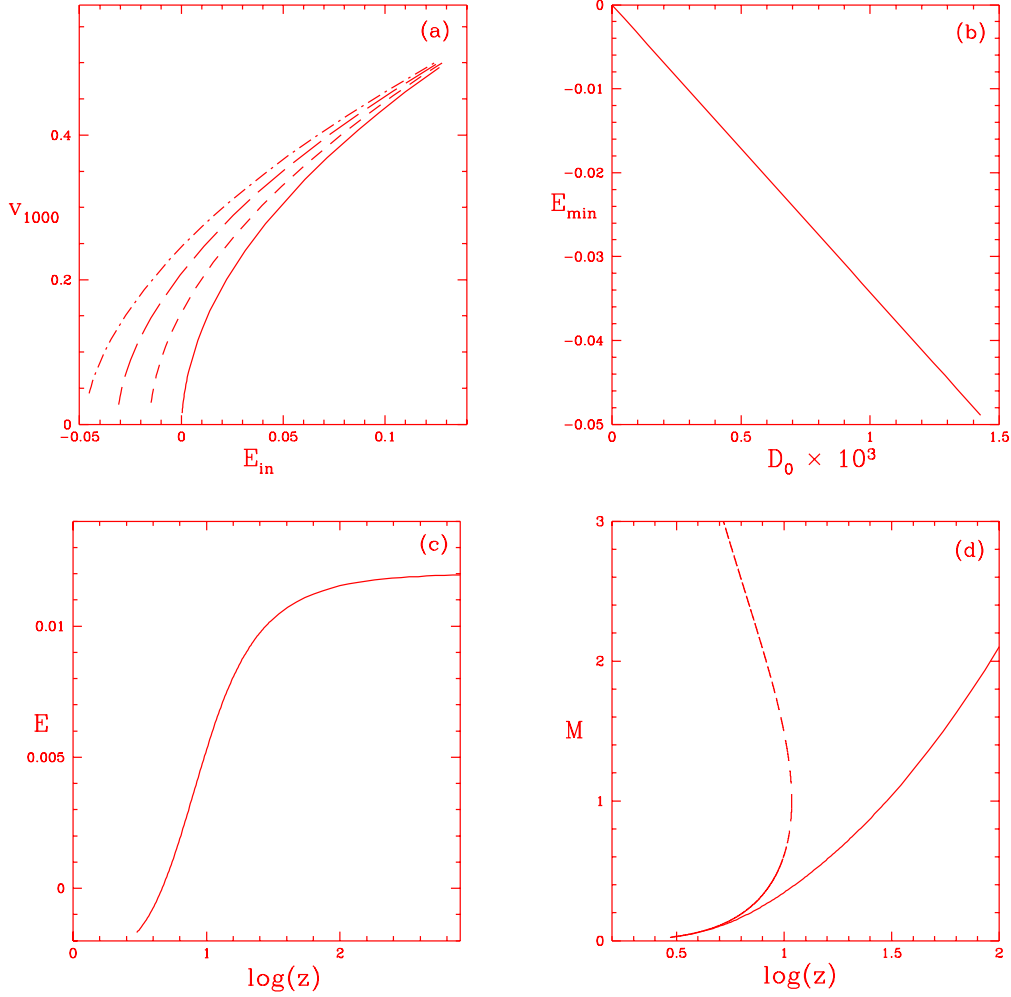


Figure 3. (a) Comparison of (v_{1000}) with initial energy (E_{in}) of the flow. Various curves represent flows acted on by RAMOD corresponding to $\dot{M}_{acc} = 15\dot{M}_{Edd}$ (dash-dot), $\dot{M}_{acc} = 10\dot{M}_{Edd}$ (long-dashed), $\dot{M}_{acc} = 5\dot{M}_{Edd}$ (short-dashed) and Bondi-type (solid) outflow. (b) Variation of the critical specific energy of the flow (E_{min}) with D_0 . (c) Variation of sp. energy E of the flow as a function of distance. RAMOD corresponds to $\dot{M}_{acc} = 5\dot{M}_{Edd}$. Initial parameter is $E_{in} = -0.002$ at $z_{in} = 3$. (d) Comparison of the variation of Mach number (M) with $\log(z)$. The solid curve represents the outflow acted on by RAMOD corresponding to $\dot{M}_{acc} = 5\dot{M}_{Edd}$. The dashed curve represents the outflow driven only by the thermal energy. Initial parameter is, $E_{in} = -0.002$ at $z_{in} = 3$ for both the cases.

initial energy of the outflow. We will come back to this equation to understand quite simply various phenomenon.

Let us now focus on the solution topologies of the outflow which is accelerated by the radiative momentum deposition. Figure 2a, shows the variation of bulk velocity (v) along the axis of symmetry with $\log(z)$. Various curves denote flows acted on by radiation corresponding to $\dot{M}_{acc} = 12\dot{M}_{Edd}$ (long-dashed), $\dot{M}_{acc} = 6\dot{M}_{Edd}$ (short-dashed) and the solid line denotes Bondi-type outflow. The initial parameters are the same in all the curves — — — $v_{in} = 0.02$ (in units of c) at $z_{in} = 2.97$ (in units of $2GM/c^2$). We see that the outflows are accelerated by radiative momentum deposition. As outflows with the same input parameters

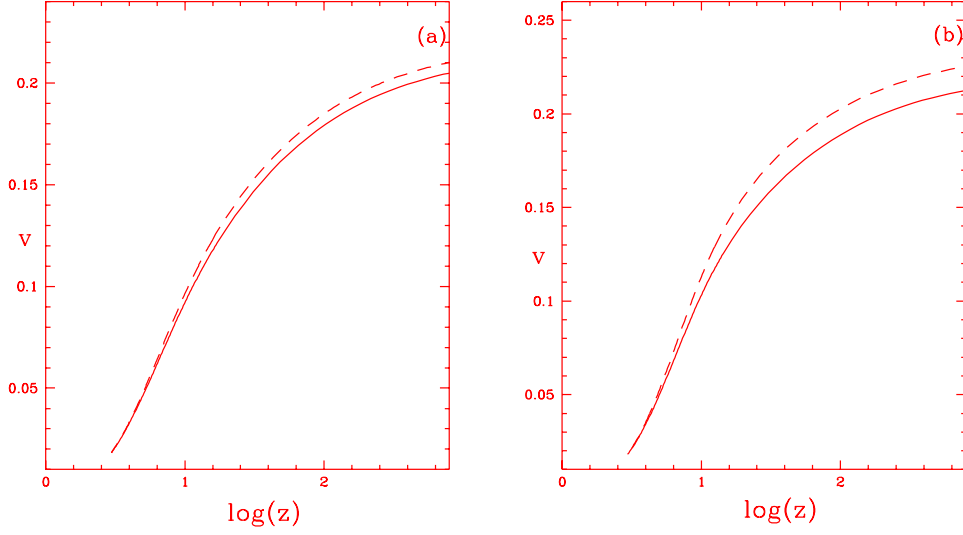


Figure 4. Comparison of velocity variation of outflows with (solid) and without (short-dashed) radiation drag. RAMOD corresponds to (a) $\dot{M}_{acc} = 5\dot{M}_{Edd}$ and (b) $\dot{M}_{acc} = 10\dot{M}_{Edd}$; $z_{in} = 2.96$, $v_{in} = 0.02$.

are accelerated to higher and higher terminal velocities, correspondingly sonic point will also come closer. Figure 2b, exhibits this phenomena where the sonic point (z_c) is plotted with non-spatial part of RAMOD [\mathcal{D}_0 ; see, Eq. (10)]. Input parameters are the same as that of Fig. 2(a).

Figure 3(a), shows the variation of v_{1000} (v at $z = 1000$) with initial energy (E_{in}) of the flow for outflows acted on by RAMOD proportional to $\dot{M}_{acc} = 15\dot{M}_{Edd}$ (dash-dot), $\dot{M}_{acc} = 10\dot{M}_{Edd}$ (long-dashed), $\dot{M}_{acc} = 5\dot{M}_{Edd}$ (short -dashed) and the Bondi-type outflow is denoted by the solid line. We see that though v_{1000} achieves a higher value, it is almost independent of RAMOD when E_{in} is high. For lower value of E_{in} , the radiative momentum deposition plays more significant role and even outflows which are initially bound are pushed to infinity as transonic outflows. From Eq. (23b) we see that if E_{in} is significantly greater than radiative work done then clearly v at large z will have a weak dependence on radiation. More over, we also see that, if LHS of Eq. (23b) is less than zero then there is no outflow. Since outflows originate close to the black hole, for cold plasma E_{in} may be negative. In that case the wind will blow if the second term in RHS of Eq. (23a) is greater than E_{in} . Thus

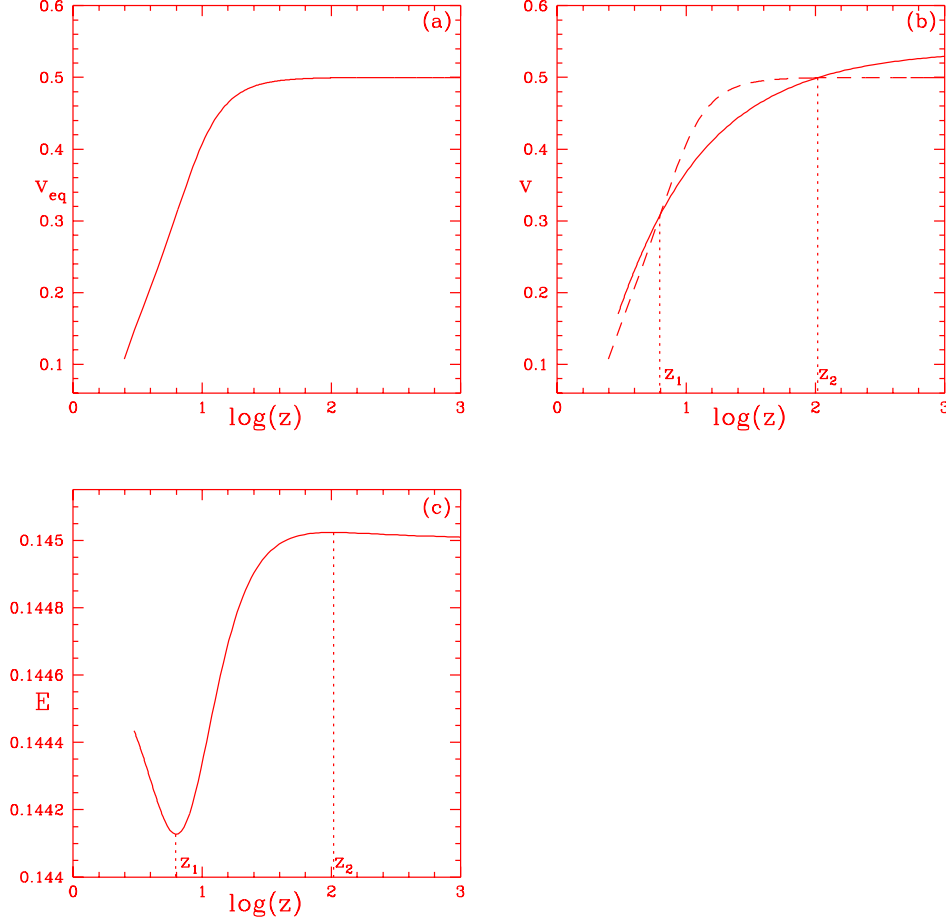


Figure 5. (a) Variation of v_{eq} as a function of z (b) Comparison of variation of bulk velocity v (solid) with v_{eq} (short-dashed) as a function of z . $v_{in} = 0.17$ at $z_{in} = 2.96$. $v_{eq}(z_{in}) < v_{in}$. The outflow topology (solid) is acted on by RAMOD corresponding to $\dot{M}_{acc} = 5\dot{M}_{Edd}$. (c) Comparison of E of the outflow in the previous case.

the critical specific initial energy E_{min} for which wind do not blow, will be obtained when these two terms are equal. Figure 3(b), shows E_{min} as a function of \mathcal{D}_0 . Outflows with E_{in} on this curve and bellow will not reach a large distance.

Figure 3(c) shows the variation of specific energy of the outflow which was initially bound, energetically speaking. Initial parameters are $E_{in} = -0.002$ (in units of c^2) at $z_{in} = 3$. Consequently, the Mach number ($M = v/a$) variation of the same solution (solid) is compared with only the thermally driven outflow (short-dashed), starting with the same initial condition we find no transonic solution at all, as is exhibited in Fig. 3(d).

We now investigate the role of radiative deceleration or the radiation-drag term i.e.,

the third term in the RHS of the momentum balance equation. The radiation drag term is proportional to both v and \mathcal{E}_z . i.e., as v increases radiation drag also increases. But if the radiation energy density increases, the drag term will increase too. This phenomenon is exhibited in Fig. 4. Figure 4(a), shows the comparison of velocity (v) variation of outflows with (solid) and without (short-dashed) radiation drag. RAMOD corresponds to $\dot{M}_{acc} = 5\dot{M}_{Edd}$ was chosen. Initial parameters are $z_{in} = 2.96$, $v_{in} = 0.02$. Figure 4(b), shows the comparison of velocity (v) variation of outflows with (solid) and without (short-dashed) radiation drag for RAMOD corresponding to $\dot{M}_{acc} = 10\dot{M}_{Edd}$. Increasing the radiation one would have expected more acceleration and hence more v_{1000} at large z . But as this also increases the radiative energy density the drag term increases and results in marginally higher v_{1000} . As radiative deceleration is proportional to both v and radiative energy density, the actual deceleration will depend on competition of both of these terms. Outflows are generated close to the black hole, radiation energy density due to the inner part of the disc is large and initial velocity of the outflow is small. Hence, initially radiation drag is not strong enough to stop the outflow from forming. As outflows achieve supersonic velocity far away from the black hole, radiation energy density falls off, thus radiation drag once again is not strong enough to make the outflow decelerate or for that matter to fall back on the disc. Now the question is what is the maximum velocity allowed before deceleration sets in. If we equate the two terms in RHS of Eq. (2), we get the equilibrium speed above which there would be deceleration. Thus,

$$v_{eq} = \frac{\mathcal{D}_z}{2\mathcal{E}_z} \quad (24)$$

Thus we see that the maximum velocity beyond which deceleration sets in depends on both \mathcal{D}_z and \mathcal{E}_z . Inside the funnel shaped region \mathcal{E}_z is large, but \mathcal{D}_z is small, as flux of radiation along z is smaller. As outflowing matter leaves the funnel \mathcal{D}_z increases compared to \mathcal{E}_z and consequently v_{eq} also increases. The dashed curve in Fig. 5(a) shows the variation of v_{eq} as a function of z . As $z \rightarrow$ large, $v_{eq} \rightarrow 0.5$. This is what Icke(1989) called as the ‘magic speed’, which for a thin disc was found to be equal to 0.451. Important thing to note is that though magic speed or in our parlance v_{eq} at large z , does not depend on the magnitude of radiation intensity from the CENBOL, but actual deceleration will. In Fig. 5(b), matter is injected with v_{in} larger than $v_{eq}(z_{in})$. In the region $z < z_1$, $v > v_{eq}$ so there is a net deceleration and the flow velocity decreases than what it should be. At $z = z_1$ $v(z_1) = v_{eq}(z_1)$ which means that the radiative force is zero. For $z_2 > z > z_1$, $v < v_{eq}$,

radiation again accelerates the flow. At $z > z_2$, again $v > v_{eq}$, the flow again decelerates but as at large z , net radiative deceleration is small, the effect is marginal. If one looks at the variation of E of such a flow, the effect is more understandable. In Fig. 5(c), specific energy of the flow [same as Fig. 5(b)] is plotted with z . In the region $z < z_1$, there is net deceleration so E decreases. At $z > z_1$, there is acceleration, hence E increases. Again at $z > z_2$, $v > v_{eq}$ which means there is radiative deceleration but as the magnitude of deceleration is small, the effect is marginal. Note however, v_{eq} is not the terminal velocity. For $z \rightarrow \infty$, $E(z) \rightarrow \frac{1}{2}v_\infty^2$. From Eq. (23) we get,

$$\frac{1}{2}v_\infty^2 = E_{in} + \int_{z_{in}}^{\infty} (\mathcal{D}_{z'} - 2v\mathcal{E}_{z'}) dz'. \quad (23c)$$

The above equation tells us that v_∞ will depend on both the net radiative work done on to the flow and also the initial energy of the flow.

Apart from looking at acceleration phenomenon we have also analyzed Eq. (18), to see if there is any possibility of multiple sonic points. But we did not find multiple sonic points. This limits the scope of formation of radiative-hydrodynamic shocks in winds, although in time dependent rotating flow, shocks can still form.

Jets originate from post-shock accretion flow close to the compact object. And hence outflows should be very hot. We have seen so far that for sufficiently hot flow radiative acceleration has a very limited effect. In fact, very close to the compact object cooling processes may be important. In the entire analysis so far, we ignored cooling processes. We know that bremsstrahlung cooling (Chattopadhyay and Chakrabarti, 2000) is a very inefficient process. Therefore in the following analysis we include the synchrotron loss due to stochastic magnetic field. We assume an equipartition magnetic field (B). The synchrotron loss ($\text{erg g}^{-1} \text{s}^{-1}$) is given by Shapiro and Teukolsky, (1983),

$$S_L = \frac{16}{3} \frac{e^2}{c} \left(\frac{eB}{m_e c} \right)^2 \left(\frac{k_B T_e}{m_e c^2} \right)^2 \frac{1}{m_p}, \quad (25)$$

where, e and m_e are the electron charge and its mass. We include this cooling term in RHS of Eq. (4). Following similar method as explained in §2, we get the sonic point condition. The first condition is the same as Eq. (19) but the second condition is given by:

$$\left[1 + \frac{(\gamma - 1)S_0}{2(z_c - 1)z_c} \right] a_c^2 - z_c \mathcal{E}_z a_c + \frac{z_c}{2} \left[\mathcal{D}_{z_c} - \frac{1}{2(z_c - 1)^2} \right] = 0, \quad (26)$$

where, $S_0 = (32e^4 \mu^2 \pi 1.4 \times 10^{17} f) / (3m_e^3 c^3 \gamma^2 M_\odot G \Theta_{out})$, where f is the outflow rate in units of \dot{M}_{Edd} .

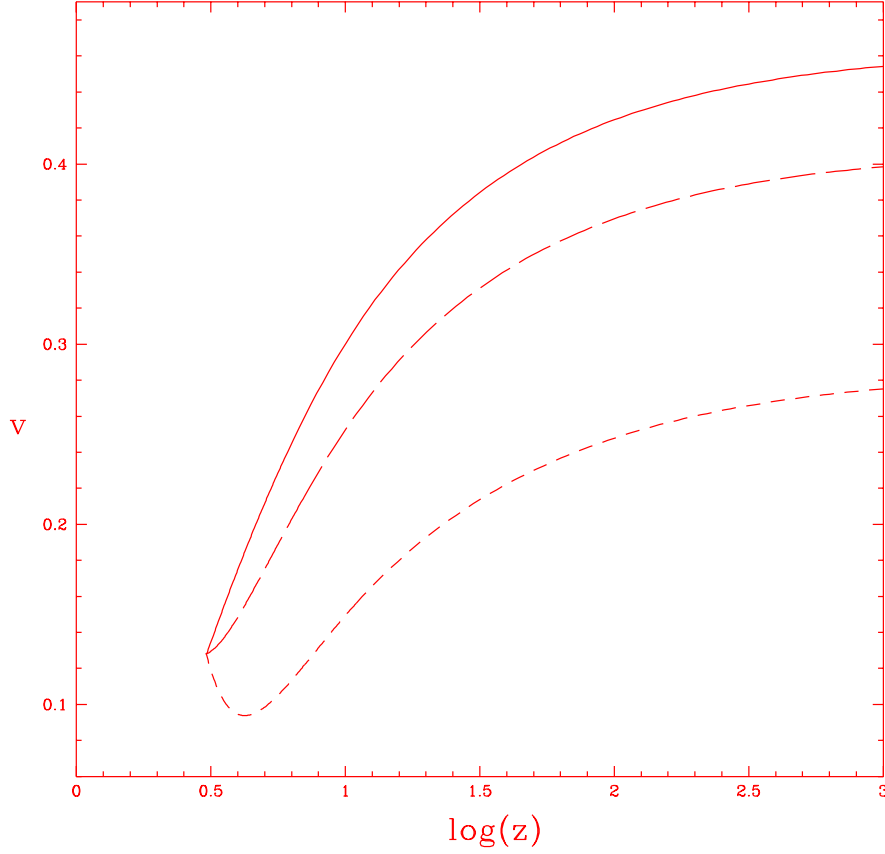


Figure 6. Solution of outflows. Initial parameter $v_{in} = 0.13c$ at $z_{in} = 3.05r_g$. RAMOD corresponds to $\dot{M}_{acc} = 6\dot{M}_{Edd}$. Variation of outflows without cooling (solid) is contrasted with outflows with cooling for $\dot{M} = 0.01\dot{M}_{Edd}$ (short-dashed) and $\dot{M} = 0.005\dot{M}_{Edd}$ (long dashed).

Even in this case there is only one positive real root for sonic point outside the horizon. Hence once again the possibility of finding a steady shock in the transonic outflow is absent. Figure 6, compares the variation of bulk velocity without (solid) cooling and with cooling for $\dot{M} = 0.01\dot{M}_{Edd}$ (short-dashed) and $\dot{M} = 0.005\dot{M}_{Edd}$ (long dashed). RAMOD corresponds to $\dot{M}_{acc} = 6\dot{M}_{Edd}$. Initial parameters are $z_{in} = 3.05$ and $v_{in} = 0.13$. We can see that synchrotron cooling produces less energetic outflow. However, synchrotron loss rate (per unit volume) depends on the number density of electrons and also the magnetic field strength. Number density of electrons depends on the mass outflow rate (\dot{M}). As we have no theoretical handle on the magnetic field, we have assumed an equipartition magnetic field (B), which depends on the density of matter and hence on \dot{M} . Thus tuning \dot{M} would also tune B . We reduce \dot{M} to $0.005\dot{M}_{Edd}$ (long-dashed) and find that compared to the solution which includes cooling

(long-dashed) with higher \dot{M} results in outflows which are more energetic. If one had a better understanding of magnetic fields around accreting black holes, then synchrotron cooling phenomenon would be worth pursuing. This is just to show cooling can be very important close to the black hole.

4 DISCUSSION AND CONCLUDING REMARKS

It has been pointed out in §1, that outflows around compact objects and especially black holes have to originate from the inner part of the accreting matter. This should, in-fact be a region which is also the source of hard photons. As a photon interacts with an electron, Eq. (1) tells us that there will be exchange of energy between them and therefore momentum. If the thermal energy of the electron is less than the photon energy it will gain energy from the photon. But this does not ensure enhancement of bulk kinetic energy of the flow. Energy gained may not enhance its kinetic energy. Intense radiation in the vicinity of the black hole would create sufficient radiative pressure on the flow to slow it down due to radiation drag. So even if one considers only cold plasma even then radiative acceleration of outflows remains a tricky issue which we have tried to resolve in the present paper.

We have witnessed that as one increases radiative momentum deposition force, one gets more and more energetic outflows, but there is no linear relation between velocity at large distance and the radiative force. This is because of radiation drag effect. Higher radiative force gives rise to a higher energy density. In principle, higher radiative force means more acceleration resulting in higher velocity, but as radiation drag term is proportional to both v and energy density, therefore the deceleration starts to be important too. Equating the first term and the second term in RHS of Eq. (2), we get the expression of equilibrium speed v_{eq} . If one considered the disc to be an infinite flat radiator, as Icke (1989) did, at large distances, the contribution to the radiative energy density remains higher compared to the flux along the axis of symmetry. This is not true when the ‘radiator’ has a special geometry as in our case. Here, within the *funnel* of the inner torus of the *finite* accretion disc, radiation received at any point on the axis is significantly larger than the flux along the axis. As one goes further and further away from the inner torus, the matter will increasingly see it as a point source. Hence the energy density will be that due to a point source. The efficiency of ‘flux-focussing’ is thus better. As a result, we get marginally higher equilibrium velocity (at a large distance) than that obtained for a thin accretion disc. Icke (1989) has

also computed magic speed for the thick disc where the temperature of the inner surface of the thick disc decreases with the height. He showed that this temperature variation of the driving surface significantly influence the value of magic speed. In our case the radiation coming out of the inner torus are inverse Comptonized photons and are not thermalized with the post-shock torus. Moreover, the CENBOL surface is an isothermal one. But the frequency averaged intensity will depend on the temperature gradient at the surface of the CENBOL. With the particular case we have taken, the temperature gradient is seen not to vary appreciably except very close to the black hole. Hence for simplicity we have taken the intensity of radiation to be uniform.

Recently Fukue and his collaborators (1999-2001) has systematically studied the radiative acceleration and collimation of jets coming out from a disc which is a combination of inner ADAF region (non luminous) and outer slim disc (luminous). First of all, the ADAF solutions are for very low accretion rate ($\dot{M}_{acc} \lesssim 10^{-5} \dot{M}_{Edd}$), so the outflow rate would be even smaller. The second fact is that the radiative collimation they have got is only because of the inner non-luminous region which does not contribute to the radial (r, ϕ, z system) flux. The situation is different in our case. The hard radiation is coming from the inner post-shock region of the disc while the soft radiation is coming from the outer region of disc. In this paper we have only considered the radiations coming from the post-shock region. Therefore we are working in a different regime.

We ignored bremsstrahlung cooling because it is a very inefficient cooling mechanism and is not likely to change the physics of outflows, qualitatively. We also have ignored cooling due to inverse-Comptonization. As we initially were interested to look at acceleration phenomenon of the radiation we had to look for cold plasma. Thus the equations of motion presented in §2. are not suitable to study the effect of inverse Comptonization. In future we will work with the most general form of radiative transfer equation in curved space time, which will include all the processes.

While calculating the specific intensity of radiation (in §2.3) coming out from the inner torus of the accretion disc, we have assumed that the radiation coming out is proportional to the gravitational energy release of in-falling matter. The conversion ratio (η) used, was that due to last stable orbit around a Schwarzschild black hole. As the Keplerian disc only extends upto the shock location (i.e., R_s in §2.4) hence η will be less than 0.06, around 0.04. Hence there is a little bit of over estimation in the calculation of the terminal velocity, though this is not going to change the result qualitatively.

Our conclusion in the present paper is that the radiation momentum deposition does accelerate matter to produce transonic outflows. Radiative acceleration increases with the accretion rate. Achievement of very high velocity is restricted because of the radiation drag term. Radiation drag term is the deceleration term and it depends on both the velocity of the flow and the energy density of radiation. As a result above some speed (v_{eq}) radiative deceleration sets in. Within the funnel of the inner post-shock torus of the accretion disc, v_{eq} depends on distance but at large distance from it the steady value attained is 0.5 i.e. fifty percent of the velocity of light. It is to be noted that v_{eq} itself is not the terminal velocity v_∞ – it becomes the terminal velocity for the flow having an infinitesimally small initial energy E_{in} [see, Eq. 23(c)]. If $E_{in} > 0$, it can be easily shown that $v_\infty^2 < v_{eq}^2 + 2E_{in}$.

Radiative-hydrodynamics changes the critical point condition, but fails to generate multiple real critical points of the equation thus eliminating the possibility of getting steady shocks in outflows. This may be related with the assumptions enforced, namely, due to the consideration of cold plasma in a *non-rotating* flow. Radiative momentum deposition increases the specific energy of the outflow to the extent of driving bound outflowing matter to infinity as transonic outflow. Synchrotron cooling results in less energetic outflow, but this effect can increase with larger outflow rate. A proper theoretical handle on the structure of magnetic field will dramatically affect the physics of outflows.

The results presented here are calculated bearing a bipolar outflow in mind. It will be noticeable that, the parameters, like the accretion and outflow rates, used here are on the higher side. This was done to study the effects in more extreme conditions. Essential features observed here will remain unaffected for the lower values of the parameters as well.

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